

Earth to Moon: energy, windows and rendezvous

Target the moving Moon, not a fixed point

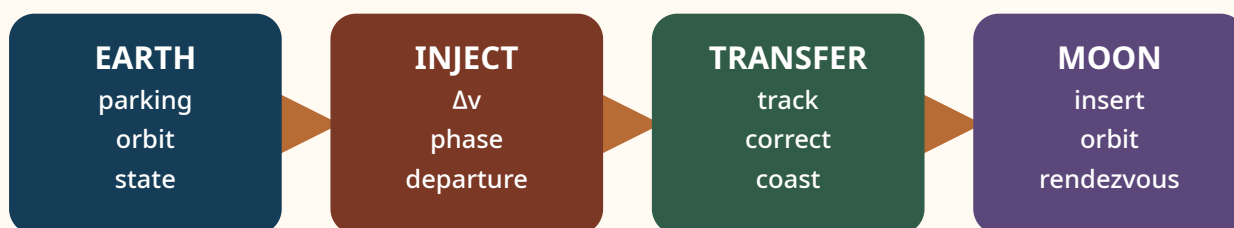
Starting question — How do departure time, lunar motion and arrival geometry combine to make a translunar trajectory work?

Intuition. The Moon moves significantly during a multi-day transfer. A correct trajectory must therefore solve timing and geometry together, not aim at the Moon's current position.

- Explain the governing physical idea before calculating.
- Name every symbol and unit used in the key relation.
- Check the result with an independent inverse, bound or order-of-magnitude test.

Earth → Moon mission

Earth → transfer → correction → lunar orbit



Every burn changes energy and future geometry.

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An Earth–Moon flight is a sequence of energy and geometry changes rather than a straight line through space.

The Moon is close enough for short communication delay yet distant enough to force orbital-mechanics reasoning. This module turns the orbit concepts of Module 05 into a complete mission story: departure, translunar transfer, correction, arrival, insertion and rendezvous. The numerical examples are deliberately educational; they teach how to close a calculation rather than reproduce a flight-certified trajectory.

1. Why a Moon mission still needs geometry and timing

The Moon moves while the spacecraft travels. Departure must therefore place the transfer trajectory where the Moon will be, not simply aim at its current visual position. Launch site, parking-orbit plane, flight time, arrival lighting and mission constraints all shape the usable opportunity. The basic discipline is to separate present position, predicted encounter position and time of flight.

A lunar launch window is not merely a calendar date. It links the parking-orbit plane, the Moon's future position and the direction of translunar injection. Earth rotates beneath the orbital plane while the Moon advances during a roughly three-day direct transfer. Mission design therefore reasons about a future state: where the target will be at encounter, not where it is at launch. A pointing error of a few tenths of a degree at injection can grow into hundreds of kilometres of miss distance near the Moon. A defensible window study propagates the state, declares the reference frame and compares candidate departure times before calling any date acceptable.

Lunar operations connect orbital mechanics to surface logistics

A crewed Earth–Moon architecture is not complete when a trajectory reaches lunar orbit. The chosen orbit changes communications coverage, eclipse duration, landing energy and the logistics of moving crew or cargo between vehicles. A low orbit can reduce descent energy but requires frequent orbital motion and careful rendezvous timing; higher or specialised orbits can improve visibility or staging while changing transfer cost. The useful lesson is to carry one budget across the whole chain: launch, translunar injection, correction, lunar arrival, staging and surface access. A manoeuvre that looks inexpensive in isolation can create a more expensive or less recoverable next step.

2. Read a delta-v budget as a list of orbital changes

Δv (“delta-v”) is a commanded change in velocity, normally expressed in m/s or km/s. It is not the spacecraft's absolute speed. A mission budget can include translunar injection, trajectory corrections, lunar orbit insertion, rendezvous and reserve.

Exercise A — close a budget

A simplified mission allocates 3.15 km/s for injection, 0.12 km/s for corrections and 0.90 km/s for insertion. Add an 8 percent margin to the nominal total.

Nominal total = 4.17 km/s. Eight percent is 0.3336 km/s. Budget = 4.5036 km/s, approximately 4.50 km/s. In real design, margin should be tied to identified uncertainty rather than used as an unexplained cushion.

A delta-v budget is a ledger of velocity changes tied to physical manoeuvres and reserves. In a teaching lunar mission, translunar injection from low Earth orbit is of the order of a few kilometres per second, while trajectory corrections may be tens of metres per second or less. Adding magnitudes is not enough: a burn can spend the planned delta-v in the wrong direction and create the wrong trajectory. Each line should therefore record reference frame, direction, execution time, expected dispersion and contingency allowance. That discipline is more valuable than memorising a single lunar number because the same method scales directly to a Mars campaign.

3. Use the rocket equation with its assumptions visible

The ideal rocket equation is $\Delta v = v_e \ln(m_0/m_f)$. v_e is effective exhaust velocity, m_0 initial mass, m_f final mass and $\ln$ the natural logarithm. It exposes the logarithmic penalty of increasing delta-v. It does not include gravity losses, finite-burn steering, structural limits or thermal constraints.

The Tsiolkovsky equation connects ideal delta-v to exhaust velocity and mass ratio: $\Delta v = v_e \ln(m_0/m_f)$. The natural logarithm, $\ln$, means propellant cost rises rapidly when more delta-v is demanded from the same propulsion system. With $v_e = 3,200$ m/s and $m_0/m_f = 2$, ideal delta-v is about $3,200 \times 0.693 = 2,218$ m/s. This ignores gravity losses, finite burns, reserves and operational constraints. Its value is diagnostic: it checks whether a proposed mass breakdown is plausible and shows why adding a few hundred metres per second of margin can produce a disproportionate increase in initial mass.

4. Flight time is an architectural choice

Faster transfers can demand more energy; slower transfers change exposure, consumables, navigation geometry and system operating time. Historic crewed lunar missions demonstrate travel on the order of days, but the useful lesson is not to memorize one duration. It is to understand that time, energy and mission risk are traded together.

Flight time trades propulsion against operations. A faster transfer can reduce exposure and the time during which systems must remain continuously healthy, yet it may require more energy or a narrower departure geometry. A low-energy transfer

can save delta-v while increasing duration, navigation workload and exposure to failures. For a crewed mission, every extra day also consumes water, oxygen, food, electrical energy and human availability. A useful trade therefore converts time into physical resources and risk. A trajectory is not inherently 'slow' or 'fast'; it is acceptable only when propulsion, life support, navigation and return strategy close together.

5. Trajectory correction buys accuracy

Tiny state errors at departure become large position errors after a long coast. Correction manoeuvres are planned because launch, burn execution and navigation are never perfect. Earlier correction can be efficient, while waiting can allow tracking data to improve the state estimate. Guidance therefore trades correction cost against knowledge.

A trajectory correction is valuable only when it is based on a credible state estimate. A 1 m/s velocity error maintained for 24 hours corresponds, in a deliberately simple straight-line estimate, to 86.4 km of position error. Real gravitational dynamics are more complex, but the scale shows why small velocity errors matter. Correcting early can reduce required delta-v; waiting may improve tracking but increase correction cost. The decision uses navigation covariance, time to the next critical event and propulsion constraints. After the burn, new measurements must demonstrate the achieved state. Executing a command is not evidence that the intended trajectory was obtained.

6. Arrival is not capture

A spacecraft approaching the Moon does not naturally remain there. To enter a bound orbit it must change relative energy, commonly through propulsion. Lunar orbit insertion is therefore an integrated event: navigation, attitude, propulsion, timing and post-burn orbit determination all need to agree.

Exercise B — thrust and burn time

A 20 kN engine acts on a 10,000 kg spacecraft. Ignoring mass change, what acceleration does it produce and how long would an ideal constant acceleration require for 800 m/s?

$a = F/m = 20,000/10,000 = 2 \text{ m/s}^2$. Ideal time $= \Delta v/a = 800/2 = 400 \text{ s}$. Real burns change mass and direction, but the arithmetic links thrust, mass and manoeuvre duration.

Approaching the Moon at the correct distance does not make the spacecraft captured. It still carries relative orbital energy and can simply fly past unless a burn or naturally bound trajectory removes enough of that energy. Lunar-orbit insertion is usually timed near close approach, where an impulsive velocity change strongly affects the resulting orbit.

Underperformance may leave a very high ellipse or an escape trajectory; overperformance can lower perilune dangerously. The mission therefore defines burn limits, cutoff criteria, fallback orbits and post-burn navigation before arrival. Capture is a controlled transition between orbital-energy states, not a geographic crossing.

7. Rendezvous begins with orbital phasing

One spacecraft cannot simply point at another and accelerate toward it. The chaser changes orbit to alter relative phase, then reduces closure rate as range decreases. Navigation transitions from kilometres to hundreds of metres to proximity operations. The sensor suite and keep-out rules evolve at each scale.

Orbital rendezvous is a problem of relative position and relative velocity. Two spacecraft can pass through the same point with incompatible velocities and never be in a safe rendezvous condition. The sequence first brings orbital planes into acceptable alignment, then adjusts phase so target and chaser reach the same neighbourhood at the right time. At close range, relative motion is counter-intuitive: accelerating forward may initially alter orbital altitude and period rather than simply 'catch up'. Training must therefore separate relative navigation, manoeuvre planning, approach zones and abort rules. A safe rendezvous always includes a passive or actively commanded escape path.

8. Timing error can become position error

For intuition, $\Delta \mathbf{x} \approx \mathbf{v} \times \Delta t$. At a relative speed of 1,600 m/s, a one-second timing error corresponds to 1.6 km along track. $\Delta \mathbf{x}$ is displacement, $\mathbf{v}$ speed and Δt time error. Orbital propagation is more complex, but the estimate explains why timekeeping

belongs to navigation.

Timing error becomes geometry error. If a critical event is shifted by 30 seconds while local orbital speed is 1.6 km/s, the vehicle travels roughly 48 km along its path during that interval. Distance $\approx$ speed $\times$ time is not a complete orbital propagation, but it immediately reveals the scale. Clocks, state estimation and command execution need a coherent time reference. Software must also distinguish measurement time from reception time. That habit becomes even more important on Mars missions, where Earth messages arrive minutes after the state they describe.

9. Failure scenario: lunar insertion shuts down early

An early engine cutoff can leave a different orbit or a lunar flyby trajectory. The first recovery action is not “burn again immediately.” The team reconstructs the achieved state, propellant reserve, communication geometry and collision or escape risk. A second manoeuvre is designed only after the actual trajectory is understood.

An interrupted lunar-insertion burn should be treated as a branching set of future trajectories, not as a binary failure. The team first estimates the impulse actually delivered and propagates the resulting state: bound lunar ellipse, flyby, impact possibility or return toward Earth’s sphere of influence. A second improvised burn should not begin before that state is known. Options depend on remaining propellant, tracking visibility, electrical energy and time to the next close approach. Robust mission design provides cutoff commands, safe pointing modes and trajectories that preserve decision time. Recovery begins as a navigation problem and only then becomes a propulsion problem.

10. Use the Moon to learn principles that scale to Mars

Lunar operations benefit from frequent opportunities and short communication delay. Mars adds heliocentric geometry, long delays, rare launch windows and months of cruise. Yet the reasoning pattern remains: budget delta-v, estimate state, correct uncertainty, plan arrival, preserve reserve and define failure responses before launch.

The Moon is a powerful training environment because it exposes many of the same problem families as Mars while remaining much closer. The scales, however, change radically: roughly one-second radio delay to the Moon versus many minutes to Mars, no lunar atmosphere versus a thin Martian atmosphere, days of transfer versus months, and very different rescue opportunities. The transferable content is therefore principles—margins, navigation, rendezvous, autonomy and maintenance—not copied architectures. A strong final exercise takes a valid lunar decision and identifies exactly which assumptions stop being valid when the same idea is extended to Mars.

Guided case — from injection dispersion to lunar arrival

Assume a nominal translunar injection followed by a navigation estimate showing +4 m/s along-track error and –2 m/s cross-track. The errors should not simply be added as $4 + 2$ because they are not collinear. Vector magnitude is about $\sqrt{(4^2+2^2)}=4.47$ m/s, but correction cost depends more importantly on how each component changes future arrival geometry. A cross-track error can move the lunar encounter plane more strongly than an equal along-track error. Navigation therefore propagates candidate futures and guidance selects a correction that reduces future risk rather than merely cancelling the instantaneous vector.

Mission design then reserves another correction opportunity. If the first burn is executed early, its own execution error can be observed for hours before the next decision. This separates a large early cleanup from a small late trim. Delta-v reserve must cover both, along with navigation uncertainty and possible avoidance action. A useful exercise allocates 30 m/s of correction reserve among three possible windows and explains why spending the entire reserve at the first sign of error may be poor strategy.

Success is finally proved by a post-burn state estimate whose uncertainty fits the lunar-arrival corridor, not by a telemetry flag saying the engine burned for the commanded time. That distinction between command, measurement and achieved state carries directly into later GNC and EDL modules.

11. Mini-project: build an explainable flight plan

1. Select a notional Earth parking orbit.
2. List injection, correction and insertion delta-v.

3. Add margin and state what uncertainty it covers.
4. Describe the measurements used for navigation.
5. Define a response to incomplete insertion.
6. Assign decisions between automation, crew and Earth.

The objective is a traceable chain of assumptions, not a flight-certified answer.

12. Mission lab — recover from an imperfect translunar injection

Assume post-burn navigation estimates a 6 m/s error on the velocity component that most affects lunar arrival. The disciplined response is not automatically a 6 m/s burn in the opposite direction. First propagate the error to encounter time, quantify measurement uncertainty and determine whether waiting for better tracking will reduce total correction cost. Navigation represents uncertainty and correlation; guidance then selects a correction that improves arrival geometry without consuming reserve unnecessarily.

If a teaching correction of 6 m/s is delivered at an average acceleration of 0.30 m/s², ideal burn duration is $\Delta t = \Delta v/a = 6/0.30 = 20$ s. Δt is burn time, Δv velocity change and a average acceleration. The calculation ignores changing mass, attitude slew, ignition transient and finite-burn trajectory effects. Its purpose is to check whether a command is physically plausible.

After the burn, the spacecraft does not declare the trajectory fixed merely because the command executed. Tracking is repeated, the achieved state is estimated and predicted lunar conditions are recalculated. Recovery closes only when independent navigation evidence shows that the actual trajectory is inside the intended arrival corridor.

This illustrates a general principle: manoeuvre execution and manoeuvre verification are separate functions. A propulsion telemetry flag can prove that valves opened; it does not by itself prove the correct orbital state was achieved.

13. Alternatives, limits and review questions

Earth–Moon mission design includes direct transfers, lower-energy paths with longer flight time, different lunar orbit families and architectures involving rendezvous. “Best” depends on the objective. Minimising delta-v can increase time and operational complexity; minimising time can increase propulsion demand; simplifying rendezvous can increase lander capability requirements. Trade studies need an explicit objective function rather than a generic claim of optimisation.

Before accepting a plan, ask which measurement proves the state after each major burn; what reserve exists for underperformance; what trajectory results from incomplete insertion; how communications geometry changes after a missed event; and which decisions must be available onboard. A robust flight plan includes the off-nominal future as carefully as the nominal one.

A final useful discipline is unit checking. Keep velocity in m/s or km/s consistently, mass in kg, time in seconds when using SI equations, and angles in a declared unit. Many trajectory mistakes are not failures of celestial mechanics but failures to notice that one input changed unit or reference frame.

Calculation laboratory — certified formula reasoning

This laboratory adds the zero-prerequisite reasoning chain used by the certified French Academy: read the formula, define every symbol and unit, calculate step by step, then verify the result independently.

Earth–Moon transfer: canonical mini-lessons

These mini-lessons form the module’s canonical quantitative reference. Numerical cases are teaching examples and must remain distinct from mission data.

Delta-v budget with reserve

$$\Delta v_{\text{budget}} = (\Sigma \Delta v_{\text{nominal}}) \times (1 + f_{\text{margin}})$$

1 — Concrete question

What does “ $\Delta v_{\text{budget}} = (\Sigma \Delta v_{\text{nominal}}) \times (1 + f_{\text{margin}})$ ” compute in the context of “Delta-v budget with reserve”?

2 — Intuition without symbols

The total budget sums planned manoeuvres and then adds an explicit reserve for deviations and corrections.

3 — Quantities

Δv_{budget} : final budget; $\Sigma \Delta v_{\text{nominal}}$: sum of nominal manoeuvres; f_{margin} : reserve fraction.

4 — Formula

$$\Delta v_{\text{budget}} = (\Sigma \Delta v_{\text{nominal}}) \times (1 + f_{\text{margin}})$$

5 — Read aloud

Read “ $\Delta v_{\text{budget}} = (\Sigma \Delta v_{\text{nominal}}) \times (1 + f_{\text{margin}})$ ” by naming every operation explicitly.

6 — Symbols and meaning

Δv_{budget} : final budget; $\Sigma \Delta v_{\text{nominal}}$: sum of nominal manoeuvres; f_{margin} : reserve fraction.

7 — Pronunciation

Say subscripts, exponents, Greek letters and operations explicitly so another person can reconstruct the relation without seeing the page.

8 — Units

Velocities in m/s or km/s, but the same unit throughout; f_{margin} dimensionless.

9 — Convention

Keep the same frame, sign convention, time basis and system boundary throughout the calculation.

10 — Why this operation

The sum builds nominal need, then the reserve factor increases it proportionally.

11 — Assumptions

Manoeuvres belong to the same mission scope and the reserve is applied once.

12 — Unit check

Velocities in m/s or km/s, but the same unit throughout; f_{margin} dimensionless. Verify that units reduce to the announced output quantity.

13 — Numerical case

With $3.15 + 0.12 + 0.90 = 4.17$ km/s and 8% reserve, $\Delta v_{\text{budget}} = 4.17 \times 1.08 = 4.5036$ km/s.

14 — Why the calculation works

The sum builds nominal need, then the reserve factor increases it proportionally.

15 — Algebraic check

Dividing the final budget by 1.08 must recover 4.17 km/s.

16 — Mental estimate

8% of 4.17 is about 0.33, so the total should be near 4.50 km/s.

17 — Interpretation

The budget indicates propulsive capability to reserve, not the exact trajectory.

18 — What the result does not prove

This result proves neither universal validity of the model nor quality of the inputs; it applies only to the stated scenario and assumptions.

19 — Sensitivity

A larger reserve directly increases the budget; removing a manoeuvre first lowers the nominal sum.

20 — Guided and autonomous exercises

Guided exercise. Nominal need is 4,110 m/s with 8% reserve.

Detailed guided correction — open after trying

$$\Delta v_{\text{budget}} = 4,110 \times 1.08 = 4,438.8 \text{ m/s, about } 4,439 \text{ m/s.}$$

Autonomous exercise. Nominal need is 3,900 m/s with 10% reserve.

Autonomous correction — open after trying

$$\Delta v_{\text{budget}} = 3,900 \times 1.10 = 4,290 \text{ m/s.}$$

21 — Mission decision

Reserve delta-v before freezing propellant mass and manoeuvre sequence.

Ideal stage capability

$$\Delta v = v_e \ln(m_0 / m_f)$$

1 — Concrete question

What does “ $\Delta v = v_e \ln(m_0 / m_f)$ ” compute in the context of “Ideal stage capability”?

2 — Intuition without symbols

Mass ratio and exhaust velocity determine the ideal velocity-change capability of the stage.

3 — Quantities

Δv : ideal velocity change; v_e : exhaust velocity; m_0 : pre-burn mass; m_f : post-burn mass.

4 — Formula

$$\Delta v = v_e \ln(m_0 / m_f)$$

5 — Read aloud

Read “ $\Delta v = v_e \ln(m_0 / m_f)$ ” by naming every operation explicitly.

6 — Symbols and meaning

Δv : ideal velocity change; v_e : exhaust velocity; m_0 : pre-burn mass; m_f : post-burn mass.

7 — Pronunciation

Say subscripts, exponents, Greek letters and operations explicitly so another person can reconstruct the relation without seeing the page.

8 — Units

Δv and v_e in m/s; masses in the same unit.

9 — Convention

Keep the same frame, sign convention, time basis and system boundary throughout the calculation.

10 — Why this operation

Momentum conservation for a variable-mass rocket leads to the logarithm of mass ratio.

11 — Assumptions

Ideal model without losses and with representative v_e .

12 — Unit check

Δv and v_e in m/s; masses in the same unit. Verify that units reduce to the announced output quantity.

13 — Numerical case

With $v_e = 3,200 \text{ m/s}$ and $m_0/m_f = 2$, $\Delta v = 3,200 \times \ln(2) \approx 2,218 \text{ m/s}$.

14 — Why the calculation works

Momentum conservation for a variable-mass rocket leads to the logarithm of mass ratio.

15 — Algebraic check

$\exp(\Delta v/v_e)$ must recover mass ratio 2.

16 — Mental estimate

$\ln(2) \approx 0.69$, so the result should be about 70% of 3,200 m/s.

17 — Interpretation

This ideal capability must be compared with the Earth–Moon budget including losses and reserves.

18 — What the result does not prove

This result proves neither universal validity of the model nor quality of the inputs; it applies only to the stated scenario and assumptions.

19 — Sensitivity

Increasing mass ratio helps logarithmically; increasing v_e acts linearly.

20 — Guided and autonomous exercises

Guided exercise. $v_e = 3,300 \text{ m/s}$, $m_0/m_f = 2.2$.

Detailed guided correction — open after trying

$\ln(2.2) \approx 0.7885$; $\Delta v \approx 3,300 \times 0.7885 = 2,602 \text{ m/s}$.

Autonomous exercise. $v_e = 3,400 \text{ m/s}$, $m_0/m_f = 2.5$.

Autonomous correction — open after trying

$\ln(2.5) \approx 0.9163$; $\Delta v \approx 3,116 \text{ m/s}$.

21 — Mission decision

Verify that ideal capability covers the reserved budget before freezing the stage.

Average acceleration during thrust

$a = F / m$

1 — Concrete question

What does “ $a = F / m$ ” compute in the context of “Average acceleration during thrust”?

2 — Intuition without symbols

The same thrust accelerates a lighter vehicle more than a heavier one.

3 — Quantities

a : acceleration; F : useful net force; m : instantaneous mass.

4 — Formula

$a = F / m$

5 — Read aloud

Read “ $a = F / m$ ” by naming every operation explicitly.

6 — Symbols and meaning

a : acceleration; F : useful net force; m : instantaneous mass.

7 — Pronunciation

Say subscripts, exponents, Greek letters and operations explicitly so another person can reconstruct the relation without seeing the page.

8 — Units

a in m/s^2 ; F in N ; m in kg .

9 — Convention

Keep the same frame, sign convention, time basis and system boundary throughout the calculation.

10 — Why this operation

Newton’s second law directly relates net force, mass and acceleration.

11 — Assumptions

Representative average force and mass consistent with the phase.

12 — Unit check

a in m/s^2 ; F in N ; m in kg . Verify that units reduce to the announced output quantity.

13 — Numerical case

With $F = 20,000 \text{ N}$ and $m = 10,000 \text{ kg}$, $a = 20,000/10,000 = 2.00 \text{ m/s}^2$.

14 — Why the calculation works

Newton’s second law directly relates net force, mass and acceleration.

15 — Algebraic check

$m \cdot a$ must recover $20,000 \text{ N}$.

16 — Mental estimate

20 kN on 10 t gives about 2 m/s^2 .

17 — Interpretation

Acceleration then supports burn-time and correction-capability estimates.

18 — What the result does not prove

This result proves neither universal validity of the model nor quality of the inputs; it applies only to the stated scenario and assumptions.

19 — Sensitivity

At fixed force, lower mass increases a inversely.

20 — Guided and autonomous exercises

Guided exercise. $F = 30,000 \text{ N}$, $m = 12,000 \text{ kg}$.

Detailed guided correction — open after trying

$$a = 30,000/12,000 = 2.50 \text{ m/s}^2.$$

Autonomous exercise. $F = 18,000 \text{ N}$, $m = 9,000 \text{ kg}$.

Autonomous correction — open after trying

$$a = 18,000/9,000 = 2.00 \text{ m/s}^2.$$

21 — Mission decision

Keep acceleration compatible with pointing, structure and rendezvous.

Ideal correction duration

$$\Delta t = \Delta v / a$$

1 — Concrete question

What does " $\Delta t = \Delta v / a$ " compute in the context of "Ideal correction duration"?

2 — Intuition without symbols

With known average acceleration, required time is demanded velocity change divided by that acceleration.

3 — Quantities

Δt : duration; Δv : velocity change; a : average acceleration.

4 — Formula

$$\Delta t = \Delta v / a$$

5 — Read aloud

Read " $\Delta t = \Delta v / a$ " by naming every operation explicitly.

6 — Symbols and meaning

Δt : duration; Δv : velocity change; a : average acceleration.

7 — Pronunciation

Say subscripts, exponents, Greek letters and operations explicitly so another person can reconstruct the relation without seeing the page.

8 — Units

Δt in s; Δv in m/s; a in m/s^2 .

9 — Convention

Keep the same frame, sign convention, time basis and system boundary throughout the calculation.

10 — Why this operation

Acceleration is velocity change per unit time, so the relation is the direct inverse.

11 — Assumptions

Average acceleration assumed constant over the interval.

12 — Unit check

Δt in s; Δv in m/s; a in m/s^2 . Verify that units reduce to the announced output quantity.

13 — Numerical case

For $\Delta v = 800 \text{ m/s}$ and $a = 2 \text{ m/s}^2$, $\Delta t = 800/2 = 400 \text{ s}$.

14 — Why the calculation works

Acceleration is velocity change per unit time, so the relation is the direct inverse.

15 — Algebraic check

$a \cdot \Delta t$ must recover 800 m/s .

16 — Mental estimate

800 divided by 2 gives 400 s, about 6.7 min.

17 — Interpretation

Ideal duration excludes thrust ramp-up, shutdown and mass variation.

18 — What the result does not prove

This result proves neither universal validity of the model nor quality of the inputs; it applies only to the stated scenario and assumptions.

19 — Sensitivity

Doubling a halves time at fixed Δv .

20 — Guided and autonomous exercises

Guided exercise. $\Delta v = 6 \text{ m/s}$ and $a = 0.30 \text{ m/s}^2$.

Detailed guided correction — open after trying

$$\Delta t = 6/0.30 = 20 \text{ s.}$$

Autonomous exercise. $\Delta v = 12 \text{ m/s}$ and $a = 0.40 \text{ m/s}^2$.

Autonomous correction — open after trying

$$\Delta t = 12/0.40 = 30 \text{ s.}$$

21 — Mission decision

Verify that the manoeuvre window is long enough to execute the correction.

Position error from timing

$$\Delta x \approx v \times \Delta t$$

1 — Concrete question

What does “ $\Delta x \approx v \times \Delta t$ ” compute in the context of “Position error from timing”?

2 — Intuition without symbols

A timing error turns relative speed into approximate position error.

3 — Quantities

Δx : displacement or position error; v : representative relative speed; Δt : timing error or interval.

4 — Formula

$$\Delta x \approx v \times \Delta t$$

5 — Read aloud

Read “ $\Delta x \approx v \times \Delta t$ ” by naming every operation explicitly.

6 — Symbols and meaning

Δx : displacement or position error; v : representative relative speed; Δt : timing error or interval.

7 — Pronunciation

Say subscripts, exponents, Greek letters and operations explicitly so another person can reconstruct the relation without seeing the page.

8 — Units

Δx in m or km; v in m/s or km/s; Δt in s, with consistent units.

9 — Convention

Keep the same frame, sign convention, time basis and system boundary throughout the calculation.

10 — Why this operation

Distance travelled at constant speed is speed multiplied by time.

11 — Assumptions

Local near-constant-speed approximation; this is not full orbital propagation.

12 — Unit check

Δx in m or km; v in m/s or km/s; Δt in s, with consistent units. Verify that units reduce to the announced output quantity.

13 — Numerical case

At $v = 1.0 \text{ km/s}$ and $\Delta t = 30 \text{ s}$, $\Delta x \approx 1.0 \times 30 = 30 \text{ km}$.

14 — Why the calculation works

Distance travelled at constant speed is speed multiplied by time.

15 — Algebraic check

$\Delta x/v$ must recover 30 s.

16 — Mental estimate

One second of error at 1 km/s already means about 1 km.

17 — Interpretation

This mental check shows why clock and state estimation are critical to rendezvous.

18 — What the result does not prove

This result proves neither universal validity of the model nor quality of the inputs; it applies only to the stated scenario and assumptions.

19 — Sensitivity

Position error grows linearly with relative speed and timing error.

20 — Guided and autonomous exercises

Guided exercise. $v = 0.8$ km/s and $\Delta t = 45$ s.

Detailed guided correction — open after trying

$$\Delta x \approx 0.8 \times 45 = 36 \text{ km.}$$

Autonomous exercise. $v = 1.2$ km/s and $\Delta t = 20$ s.

Autonomous correction — open after trying

$$\Delta x \approx 1.2 \times 20 = 24 \text{ km.}$$

21 — Mission decision

Size timing precision and correction capability before terminal rendezvous.

Magnitude of a vector correction

$$||\Delta v|| = \sqrt{(\Delta v_1^2 + \Delta v_2^2)}$$

1 — Concrete question

What does “ $||\Delta v|| = \sqrt{(\Delta v_1^2 + \Delta v_2^2)}$ ” compute in the context of “Magnitude of a vector correction”?

2 — Intuition without symbols

Two perpendicular components combine through Pythagoras to give total correction-vector magnitude.

3 — Quantities

$||\Delta v||$: total magnitude; Δv_1 and Δv_2 : orthogonal components.

4 — Formula

$$||\Delta v|| = \sqrt{(\Delta v_1^2 + \Delta v_2^2)}$$

5 — Read aloud

Read “ $||\Delta v|| = \sqrt{(\Delta v_1^2 + \Delta v_2^2)}$ ” by naming every operation explicitly.

6 — Symbols and meaning

$||\Delta v||$: total magnitude; Δv_1 and Δv_2 : orthogonal components.

7 — Pronunciation

Say subscripts, exponents, Greek letters and operations explicitly so another person can reconstruct the relation without seeing the page.

8 — Units

All components and magnitude in m/s.

9 — Convention

Keep the same frame, sign convention, time basis and system boundary throughout the calculation.

10 — Why this operation

The Euclidean norm of an orthogonal vector is the square root of the sum of squares.

11 — Assumptions

Components expressed in the same orthonormal frame.

12 — Unit check

All components and magnitude in m/s. Verify that units reduce to the announced output quantity.

13 — Numerical case

With 4 m/s and 2 m/s, $||\Delta v|| = \sqrt{4^2 + 2^2} = \sqrt{20} = 4.47$ m/s.

14 — Why the calculation works

The Euclidean norm of an orthogonal vector is the square root of the sum of squares.

15 — Algebraic check

Magnitude must be at least as large as the largest component, here 4 m/s.

16 — Mental estimate

Components 4 and 2 give a result a little above 4 but below 6.

17 — Interpretation

Magnitude measures total cost while direction determines orbital effect.

18 — What the result does not prove

This result proves neither universal validity of the model nor quality of the inputs; it applies only to the stated scenario and assumptions.

19 — Sensitivity

Increasing one component increases magnitude, but not linearly while the other remains nonzero.

20 — Guided and autonomous exercises

Guided exercise. Components 3 m/s and 4 m/s.

Detailed guided correction — open after trying

$$||\Delta v|| = \sqrt{9+16} = 5.00 \text{ m/s.}$$

Autonomous exercise. Components 5 m/s and 12 m/s.

Autonomous correction — open after trying

$$||\Delta v|| = \sqrt{25+144} = 13.0 \text{ m/s.}$$

21 — Mission decision

Reserve total magnitude while preserving the direction required for the correction.

Arrival interception condition

$$||r_{\text{vehicle}}(t_{\text{arrival}}) - r_{\text{Moon}}(t_{\text{arrival}})|| \leq \epsilon_{\text{navigation}}$$

1 — Concrete question

What does “ $||r_{\text{vehicle}}(t_{\text{arrival}}) - r_{\text{Moon}}(t_{\text{arrival}})|| \leq \epsilon_{\text{navigation}}$ ” compute in the context of “Arrival interception condition”?

2 — Intuition without symbols

Interception requires more than being at the right place: vehicle and target must be close enough at the same time.

3 — Quantities

r_{vehicle} and r_{Moon} : positions at the same instant; t_{arrival} : common arrival instant at which both positions are evaluated; $\epsilon_{\text{navigation}}$: allowable separation tolerance.

4 — Formula

$$||r_{\text{vehicle}}(t_{\text{arrival}}) - r_{\text{Moon}}(t_{\text{arrival}})|| \leq \epsilon_{\text{navigation}}$$

5 — Read aloud

Read “ $||r_{\text{vehicle}}(t_{\text{arrival}}) - r_{\text{Moon}}(t_{\text{arrival}})|| \leq \epsilon_{\text{navigation}}$ ” by naming every operation explicitly.

6 — Symbols and meaning

r_{vehicle} and r_{Moon} : positions at the same instant; t_{arrival} : common arrival instant at which both positions are evaluated; $\epsilon_{\text{navigation}}$: allowable separation tolerance.

7 — Pronunciation

Say subscripts, exponents, Greek letters and operations explicitly so another person can reconstruct the relation without seeing the page.

8 — Units

Positions and tolerance in the same distance unit.

9 — Convention

Keep the same frame, sign convention, time basis and system boundary throughout the calculation.

10 — Why this operation

The norm of position difference directly measures rendezvous separation.

11 — Assumptions

Same frame, same epoch, and tolerance defined before the test.

12 — Unit check

Positions and tolerance in the same distance unit. Verify that units reduce to the announced output quantity.

13 — Numerical case

If predicted arrival separation is 4 km with a 5 km tolerance, margin is $5 - 4 = 1$ km: the geometric criterion is met.

14 — Why the calculation works

The norm of position difference directly measures rendezvous separation.

15 — Algebraic check

Separation above tolerance must fail the test even if arrival times are close.

16 — Mental estimate

4 km against a 5 km tolerance leaves 1 km of geometric margin.

17 — Interpretation

This criterion is a decision gate; it does not replace relative-velocity and attitude constraints.

18 — What the result does not prove

This result proves neither universal validity of the model nor quality of the inputs; it applies only to the stated scenario and assumptions.

19 — Sensitivity

Reducing ϵ tightens the criterion; larger state error increases predicted separation.

20 — Guided and autonomous exercises

Guided exercise. Predicted separation 6 km, tolerance 5 km.

Detailed guided correction — open after trying

Separation exceeds tolerance by $6 - 5 = 1$ km: criterion not met; trajectory or timing must be corrected.

Autonomous exercise. Separation 2.5 km, tolerance 3 km.

Autonomous correction — open after trying

Geometric margin is $3 - 2.5 = 0.5$ km: criterion met.

21 — Mission decision

Proceed to terminal rendezvous only when position, relative velocity and navigation meet their respective gates.

Mission reasoning lab — target a moving Moon

Scenario. A spacecraft leaves a high Earth orbit for a transfer lasting about 72 hours. The Moon moves roughly 13.2 degrees per day relative to the stars, so during three days it advances close to 40 degrees along its orbit. The exact mission uses numerical ephemerides, but this simple estimate is enough to expose the central idea: aiming at the Moon's current position would be a gross targeting error.

1. Synchronize time before comparing position

Define a common arrival epoch first. Propagate the Moon to that epoch and propagate the spacecraft trajectory to the same epoch. Only then compare position vectors. Mixing a spacecraft state at one time with a lunar state at another can produce a perfectly calculated but physically meaningless miss distance.

2. Make the frame explicit

A position vector is never “just a position.” It is expressed in a coordinate frame with a defined origin and orientation. Earth-centred inertial, rotating Earth-Moon and Moon-centred frames can all be useful, but subtracting vectors from different frames without transformation is invalid. The same discipline applies to velocity.

3. Use a coarse angular estimate as an independent check

At about 13.2 degrees per day, the Moon moves roughly 39.6 degrees in 72 hours. If a detailed targeting solution predicts essentially zero angular advance over that interval, the ephemeris time, units or frame definition is probably wrong. This mental check does not replace propagation; it catches category errors before they contaminate later calculations.

4. Understand sphere-of-influence switching

Patched-conic analysis often models an Earth-dominated leg followed by a Moon-dominated leg. The sphere-of-influence boundary is a convenient approximation, not a physical surface where gravity suddenly changes. State position and velocity must remain continuous through the switch. If the model creates a jump in the spacecraft state, the bookkeeping is wrong even if each local conic looks reasonable.

5. Inverse problem — correct the arrival time

Suppose the spacecraft would reach the intended lunar distance six hours too early. A first reasoning question is not immediately “what burn magnitude fixes it?” but “how far will the Moon move in six hours?” At 13.2 degrees/day, six hours correspond to about 3.3 degrees of lunar motion. That angular displacement establishes the scale of the targeting error and helps determine whether a small mid-course correction is plausible.

6. Capture versus flyby

Reaching the Moon is not the same as being captured by it. A hyperbolic arrival carries positive Moon-relative orbital energy. Unless propulsion or another mechanism removes enough energy, the spacecraft will fly past and leave. A lunar-orbit-insertion burn therefore belongs to a different energy question than translunar injection.

7. Free-return as a resilience concept

A free-return-like trajectory deliberately uses lunar gravity so that a missed or cancelled lunar manoeuvre still sends the spacecraft back toward Earth. It is not “no-control flight”: navigation and small corrections remain important. The design value is that a major propulsion failure does not automatically remove the path home.

Decision check

Before accepting a translunar targeting solution, verify: common epoch, common reference frame, plausible lunar angular advance, continuous patched-conic state, stated arrival speed, stated capture strategy and a quantified miss corridor. These checks turn a trajectory plot into an auditable mission argument.

Navigation and contingency judgement

Arrival-state uncertainty. A targeting solution is not a single geometric line. Launch injection error, manoeuvre execution error, tracking noise and force-model uncertainty create a cloud of possible arrival states. Mission design therefore propagates covariance or representative dispersions and allocates correction capability against them.

Perilune matters. Two trajectories can both “reach the Moon” but have radically different perilune altitude and velocity. Those quantities drive capture burn, occultation, surface visibility and collision risk. A useful lunar-arrival statement always includes more than miss distance.

Loss-of-burn contingency. If lunar-orbit insertion fails, the prior trajectory geometry determines whether the spacecraft escapes safely, impacts the Moon or returns toward Earth. Designing the approach with failure cases in mind turns trajectory mechanics into crew-survival engineering.

Operational discipline. Every manoeuvre should carry a latest safe execution time, navigation requirement, expected correction magnitude and post-burn verification plan. A trajectory is not operationally ready until the team knows how it will detect a bad burn and what action remains possible afterward.

Zero-prerequisite concepts

These concept cards define the vocabulary that a first-time learner must master before using it operationally.

sphere of influence

Definition. A sphere of influence is an approximation that identifies the region where one body's gravity dominates a patched-conic trajectory model.

Example. Mission design may switch from an Earth-centred approximation to a Moon-centred one near the Moon.

Pitfall. It is a modelling boundary, not a physical wall in space.

Mental check. Crossing the chosen boundary should not create a discontinuity in the actual spacecraft state.

Guided exercise — sphere of influence

Situation to recognize. A sphere of influence is an approximation that identifies the region where one body's gravity dominates a patched-conic trajectory model.

Check requested. Crossing the chosen boundary should not create a discontinuity in the actual spacecraft state.

Error to reject. It is a modelling boundary, not a physical wall in space.

Reasoned solution

Precise meaning

A sphere of influence is an approximation that identifies the region where one body's gravity dominates a patched-conic trajectory model.

Case test

Crossing the chosen boundary should not create a discontinuity in the actual spacecraft state.

Excluded pitfall

It is a modelling boundary, not a physical wall in space.

Operational consequence

Use this check before accepting a result in mission design: Crossing the chosen boundary should not create a discontinuity in the actual spacecraft state.

Quantification

sphere of influence: use the unit or dimension defined by the physical quantity; if the concept is qualitative, do not invent a numerical unit.

Verification

sphere of influence: compare the conclusion with the mental check and the stated pitfall.

translunar injection

Definition. Translunar injection is the burn or manoeuvre that places a spacecraft from Earth orbit onto a trajectory reaching the Moon.

Example. Apollo missions used a major propulsion event after parking orbit to begin the lunar transfer.

Pitfall. It is not the same as lunar-orbit insertion, which occurs near the Moon.

Mental check. A larger departure energy generally changes arrival speed and timing as well as propellant use.

Guided exercise — translunar injection

Situation to recognize. Translunar injection is the burn or manoeuvre that places a spacecraft from Earth orbit onto a trajectory reaching the Moon.

Check requested. A larger departure energy generally changes arrival speed and timing as well as propellant use.

Error to reject. It is not the same as lunar-orbit insertion, which occurs near the Moon.

Reasoned solution

Precise meaning

Translunar injection is the burn or manoeuvre that places a spacecraft from Earth orbit onto a trajectory reaching the Moon.

Case test

A larger departure energy generally changes arrival speed and timing as well as propellant use.

Excluded pitfall

It is not the same as lunar-orbit insertion, which occurs near the Moon.

Operational consequence

Use this check before accepting a result in mission design: A larger departure energy generally changes arrival speed and timing as well as propellant use.

Quantification

translunar injection: use the unit or dimension defined by the physical quantity; if the concept is qualitative, do not invent a numerical unit.

Verification

translunar injection: compare the conclusion with the mental check and the stated pitfall.

lunar capture

Definition. Lunar capture is the process of becoming gravitationally bound to the Moon, usually by reducing Moon-relative energy with a burn or other mechanism.

Example. A spacecraft arriving hyperbolically can burn near perilune to enter lunar orbit.

Pitfall. Simply entering the Moon's vicinity does not automatically mean capture.

Mental check. If no energy is removed from a hyperbolic arrival, the spacecraft normally departs again.

Guided exercise — lunar capture

Situation to recognize. Lunar capture is the process of becoming gravitationally bound to the Moon, usually by reducing Moon-relative energy with a burn or other mechanism.

Check requested. If no energy is removed from a hyperbolic arrival, the spacecraft normally departs again.

Error to reject. Simply entering the Moon's vicinity does not automatically mean capture.

Reasoned solution

Precise meaning

Lunar capture is the process of becoming gravitationally bound to the Moon, usually by reducing Moon-relative energy with a burn or other mechanism.

Case test

If no energy is removed from a hyperbolic arrival, the spacecraft normally departs again.

Excluded pitfall

Simply entering the Moon's vicinity does not automatically mean capture.

Operational consequence

Use this check before accepting a result in mission design: If no energy is removed from a hyperbolic arrival, the spacecraft normally departs again.

Quantification

lunar capture: use the unit or dimension defined by the physical quantity; if the concept is qualitative, do not invent a numerical unit.

Verification

lunar capture: compare the conclusion with the mental check and the stated pitfall.

free return

Definition. A free-return trajectory is designed so that, after a lunar encounter, gravity naturally sends the spacecraft back toward Earth without a major corrective burn.

Example. Apollo mission planning used free-return-like geometry as an important contingency option.

Pitfall. Free return does not mean zero navigation or zero correction capability.

Mental check. A robust free-return design should still bring the trajectory back near Earth after loss of the planned lunar manoeuvre.

Guided exercise — free return

Situation to recognize. A free-return trajectory is designed so that, after a lunar encounter, gravity naturally sends the spacecraft back toward Earth without a major corrective burn.

Check requested. A robust free-return design should still bring the trajectory back near Earth after loss of the planned lunar manoeuvre.

Error to reject. Free return does not mean zero navigation or zero correction capability.

Reasoned solution

Precise meaning

A free-return trajectory is designed so that, after a lunar encounter, gravity naturally sends the spacecraft back toward Earth without a major corrective burn.

Case test

A robust free-return design should still bring the trajectory back near Earth after loss of the planned lunar manoeuvre.

Excluded pitfall

Free return does not mean zero navigation or zero correction capability.

Operational consequence

Use this check before accepting a result in mission design: A robust free-return design should still bring the trajectory back near Earth after loss of the planned lunar manoeuvre.

Quantification

free return: use the unit or dimension defined by the physical quantity; if the concept is qualitative, do not invent a numerical unit.

Verification

free return: compare the conclusion with the mental check and the stated pitfall.

Beginner vocabulary checkpoint

These terms are intentionally interactive: focus or tap a term for its definition, then return to the surrounding calculation or mission decision.

- lunar orbit — An orbit in which the Moon is the primary gravitational body.
- translunar injection — Departure manoeuvre that sends a spacecraft from Earth orbit toward the Moon.
- perigee — Lowest point of an Earth-centred orbit.
- apogee — Highest point of an Earth-centred orbit.
- perilune — Lowest point of a Moon-centred orbit.
- inclination — Tilt of an orbital plane relative to a chosen reference plane.
- plane change — Manoeuvre that rotates an orbital plane and can require substantial delta-v.
- lunar orbit insertion — Burn or manoeuvre that converts an arriving lunar trajectory into a bound lunar orbit.
- free-return trajectory — Trajectory shaped so lunar gravity can return the spacecraft toward Earth without a major burn.
- sphere of influence — Approximate region where one body dominates a simplified patched-conic trajectory model.
- patched conics — Method that connects two-body trajectory arcs around different celestial bodies.
- ephemeris — Time-tagged position and velocity data for a celestial body.
- interception point — Target state where spacecraft and destination are planned to meet.
- arrival time — Epoch at which the spacecraft is expected to reach the target region.
- delta-v — Velocity change required or available for a manoeuvre.

- launch window — Time interval in which departure geometry satisfies mission constraints.
- trajectory correction manoeuvre — Small planned burn used to remove navigation and targeting error during flight.
- reference frame — Coordinate system and origin used to express position and velocity.

Sources and references

- [NASA — Apollo 11 Mission Overview](#)
- [NASA Science — Moon](#)
- [NASA Glenn — Ideal Rocket Equation](#)

Verified primary supplement: [NASA — Moon to Mars Architecture](#) · [NASA — Artemis](#)

Engineering studio — close an Earth–Moon budget

For a deliberately simplified exercise, add 3,150 m/s for translunar injection, 60 m/s for corrections and 900 m/s for the selected insertion/descent sequence. Nominal need is 4,110 m/s. An 8% reserve gives $4,110 \times 1.08 = 4,438.8$ m/s, or about 4,439 m/s. The symbol Δv is the sum of velocity changes the vehicle must be able to deliver, expressed in metres per second. Reserve is not speed that must actually be spent; it is allocated capability against navigation error, propulsion dispersion and decision delay.

The degraded case consumes half the reserve during an unexpectedly expensive correction. The student recomputes remaining capability, separates safety-critical manoeuvres from optional ones, and explains why a Δv budget alone cannot certify the mission. Available thrust, burn duration, lighting, communications and thermal limits can make a manoeuvre operationally impossible even when the energy budget appears affordable.

The Earth–Moon exercise also tests timeline margin. A correction burn that is cheap in Δv may still be unacceptable if it pushes navigation, communications or thermal operations into a constrained interval. The student therefore attaches a latest-decision time to each manoeuvre and checks whether a missed observation can be replaced before the next irreversible event. This turns the numerical reserve into an operational reserve rather than an abstract percentage.

The final record also states which reserve is protected specifically for correction after navigation updates, so nominal burns cannot silently consume it.

FR ↔ EN competency matrix

#	Français	English	Status
01	Budget Δv avec réserve	Delta-v budget with reserve	PASS
02	Capacité idéale d'un étage	Ideal stage capability	PASS
03	Accélération moyenne pendant une poussée	Average acceleration during thrust	PASS
04	Durée idéale d'une correction	Ideal correction duration	PASS
05	Erreur de position due au temps	Position error from timing	PASS
06	Norme d'une correction vectorielle	Magnitude of a vector correction	PASS
07	Condition d'interception à l'arrivée	Arrival interception condition	PASS